Pearson Edexcel

Mark Scheme (Results)

Summer 2022

Pearson Edexcel GCE
AL Further Mathematics (9FM0)
Paper 01 Core Pure Mathematics

Summer 2022
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

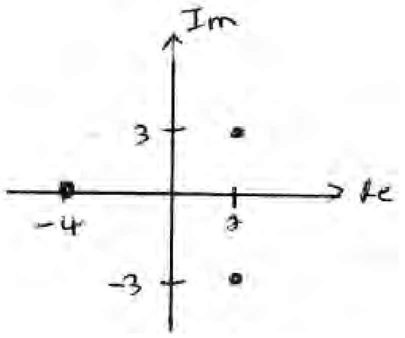
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	$2+3i$	B1	1.1b
		(1)	
(b) (i)	$z*=2+3i$ so $z+z*=4$, $zz*=13$ $z+z*+\alpha=0 \Rightarrow \alpha=\ldots$ or $\alpha zz*=-52 \Rightarrow \alpha=-\frac{52}{"13"}=\ldots$ or $z^2-(\text{sum roots})z+(\text{product roots})=0$ or $\left(z-(2+3i)\right)\left(z-(2-3i)\right)=\ldots$ $\Rightarrow (z^2-4z+13)\underline{(z+4)} \Rightarrow z=\ldots$	M1	3.1a
	$z=2\pm 3\mathrm{i},\ -4$	A1	1.1b
(ii)	$(z^2-4z+13)(z+4)$ expands the brackets to find value for a Or a = pair sum = $-4(2+3i+2-3i)+13=\ldots$ Or $f(-4)/f(2\pm 3i)=0 \Rightarrow\ldots\Rightarrow a=\ldots$	M1	1.1b
	$a=-3$	A1	2.2a
		(4)	
(c)	 	B1ft	1.1b
		(1)	
			(6 marks)

Notes:
(a) **B1:** $2 + 3i$
(b) **(i)** **M1:** A complete method to find the third root. E.g. forms the quadratic factor and uses this to find the linear factor leading to roots. Alternatively uses sum of roots = 0 or product of roots = ±52 (condone sign error) with their complex roots to find the third. Note they may have used the factor theorem to find *a* first, which is fine. If they have found *a* first, then the correct third root seen implies this mark. The method may be implied by the third root seen on the diagram. **A1:** Correct roots, all three must be clearly stated somewhere in (b), not just seen on a diagram in part (c). **(ii)** **M1:** Complete method to find a value for *a* e.g. multiplies out their quadratic and linear factors to find the coefficient of *z*, or uses pair sum, or uses factor theorem with one of the roots (may be done before finding the third root) but must reach a value for *a*. **A1:** Deduces the correct value of *a*. May be seen as the *z* coefficient in the cubic (need not be extracted, but if it is it must be correct).
(c) **B1ft:** Correctly plots all three roots following through their third root in part (b). Must be labelled with the "-4" further from *O* than 2, but don't be concerned about *x* and *y* scale. If correct look for one root on the negative real axis, with the other two symmetric about real axis in quadrants 1 and 4, but follow through their real root if positive. Accept (0,−4) labelled on the real axis in correct place as a label.

Question	Scheme	Marks	AOs
2	Solves the quadratic equation for $cosh^2 x$ e.g. $\left(8\,cosh^2 x - 9\right)\left(8\,cosh^2 + 1\right) = 0 \Rightarrow cosh^2 x = \ldots$	M1	3.1a
	$cosh^2 x = \dfrac{9}{8}\ \left\{-\dfrac{1}{8}\right\}$	A1	1.1b
	$cosh\,x = \dfrac{3}{4}\sqrt{2} \Rightarrow x = ln\left[\dfrac{3}{4}\sqrt{2} + \sqrt{\left(\dfrac{3}{4}\sqrt{2}\right)^2 - 1}\right]$ **Alternatively** $cosh\,x = \dfrac{3}{4}\sqrt{2} \Rightarrow \dfrac{1}{2}(e^x + e^{-x}) \Rightarrow e^{2x} - \dfrac{3}{2}\sqrt{2}e^x + 1 = 0$ $\Rightarrow e^x = \sqrt{2}$ or $\dfrac{\sqrt{2}}{2} \Rightarrow x = \ldots$	M1	1.1b
	$x = \pm\dfrac{1}{2}ln\,2$	A1	2.2a
		(4)	
			(4 marks)

Notes:

M1: Solves the quadratic equation for $cosh^2 x$by any valid means. If by calculator accept for reaching the positive value for $cosh^2 x$ (negative may be omitted or incorrect) but do not allow for going directly to a value for $cosh\,x$ Alternatively score a correct process leading to a value for sinh $2x$ or its square (Alt 1) or use of correct exponential form for cosh x to form and expand to an equation in e^{4x} and e^{2x} (Alt 2)

A1: Correct value for $cosh^2 x$ (ignore negative or incorrect extra roots.). In Alt 1 score for a correct value for $\sinh^2 2x$ or sinh $2x$. In Alt 2 score for a correct simplified equation in e^{4x}.

M1: For a correct method to achieve at least one value for x (from cosh ^{2} x). In the main scheme or Alt 1, takes positive square root (if appropriate) and uses the correct formula for arcosh x or arsinh x to find a value for x. (No need to see negative square root rejected.) In Alt 2 it is for solving the quadratic in e^{4x} and proceeding to find a value for x.

Alternatively uses the exponential definition for cosh x, forms and solves a quadratic for e^x leading to a value for x

A1: Deduces (both) the correct values for x and no others. Must be in the form specified.

SC Allow M0A0M1A1 for cases where a calculator was used to get the value for cosh x with no evidence if a correct method for find both values is shown.

2 Alt 1	$64\cosh^2 x\left(\cosh^2 x-1\right)-9=0 \Rightarrow 64\cosh^2 x\sinh^2 x-9=0$ $\Rightarrow 16\sinh^2 2x=9 \Rightarrow \sinh^2 2x=\frac{9}{16}$ Or $(8\,sinh\,x\,cosh\,x-3)(8\,sinh\,x\,cosh\,x+3)=0 \Rightarrow sinh\,2\,x=\pm\frac{3}{4}$	M1 A1	3.1a 1.1b
	$sinh\,2\,x=\pm\frac{3}{4} \Rightarrow x=\frac{1}{2}ln\left[\pm\frac{3}{4}+\sqrt{\frac{9}{16}+1}\right]$ (or use exponentials, or proceed via cosh4x)	M1	1.1b
	$x=\pm\frac{1}{2}ln\,2$	A1	2.2a
		(4)	
2 Alt 2	$64\left(\frac{e^x+e^{-x}}{2}\right)^4-64\left(\frac{e^x+e^{-x}}{2}\right)^2-9=0 \Rightarrow$ $4(e^{4x}+4e^{2x}+6+4e^{-2x}+e^{-4x})-16(e^{2x}+2+e^{-2x})-9=0$	M1	3.1a
	$4e^{4x}-17+4e^{-4x}=0$	A1	1.1b
	$(4e^{4x}-1)(1-4e^{-4x})=0 \Rightarrow e^{4x}=\ldots \Rightarrow x=\ldots$	M1	1.1b
	$x=\pm\frac{1}{2}ln\,2$	A1	2.2a
		(4)	

Question	Scheme	Marks	AOs
3(a)	$\frac{dy}{dx} + y\tan x = e^{2x}\cos x$ $\text{IF} = e^{\int \tan x\, dx} = e^{\ln\sec x} = \sec x \Rightarrow \sec x\frac{dy}{dx} + y\sec x\tan x = e^{2x}$ $\Rightarrow y\sec x = \int e^{2x}\ dx$	M1	3.1a
	$y\sec x = \frac{1}{2}e^{2x}(+c)$	A1	1.1b
	$y = \left(\frac{1}{2}e^{2x} + c\right)\cos x$	A1	1.1b
		(3)	
(b)	$x = 0,\ y = 3 \Rightarrow c = \ldots\{2.5\}$	M1	3.1a
	$y = \left(\frac{1}{2}e^{2x} + \frac{5}{2}\right)\cos x = 0 \Rightarrow \cos x = 0 \Rightarrow x = \ldots$	M1	1.1b
	$x = \frac{\pi}{2}$	A1	1.1b
		(3)	
			(6 marks)

Notes:

(a)

M1: Finds the integrating factor and attempts the solution of the differential equation.

Look for I.F. $= e^{\int \tan x dx} \Rightarrow y \times$ 'their I.F.' $= \int e^{2x}\cos x \times$ 'their I.F.' dx

A1: Correct solution condone missing + c

A1: Correct general solution, Accept equivalents of the form $y = f(x)$, such as $y = \frac{e^{2x}}{2\sec x} + \frac{c}{\sec x}$

(b)

M1: Uses $x = 0\ \ y = 3$ to find the constant of integration. Allow if done as part of part (a) and allow for their answer to (a) as long as it has a constant of integration to find.

M1: Sets $y = 0$ in an equation of the form $y = (Ae^{2x} + c)\cos x$ (oe) where A is 1, 2 or $\frac{1}{2}$, with their c or constant c and makes a valid attempt to solve the equation to find a value for x. (Allow even if the constant of integration has not been found).

A1: Depends on both M's. Awrt 1.57 or $\frac{\pi}{2}$ only. There must have been an attempt to find the constant of integration, but allow from a correct answer to (a) as long as a positive value for c has been found (can be scored from implicit form).

Question	Scheme	Marks	AOs
4(a)	Applies $ln\left(\frac{r+1}{r-1}\right) = ln(r+1) - ln(r-1)$ to the problem in order to apply differences.	M1	3.1a
	$\sum_{r=2}^{n}(ln(r+1) - ln(r-1))$ $= (ln(3) - ln(1)) + (ln(4) - ln(2)) + (ln(5) - ln(3)) + \ldots$ $+(ln(n) - ln(n-2)) + (ln(n+1) - ln(n-1))$	dM1	1.1b
	$ln(n) + ln(n+1) - ln\,2$	A1	1.1b
	$ln\left(\frac{n(n+1)}{2}\right)$ * cso	A1 *	2.1
		(4)	
(b)	$\sum_{r=51}^{100} ln\left(\frac{r+1}{r-1}\right) = \sum_{r=2}^{100} ln\left(\frac{r+1}{r-1}\right) - \sum_{r=2}^{50} ln\left(\frac{r+1}{r-1}\right)$ $= ln\left(\frac{100\times 101}{2}\right) - ln\left(\frac{50\times 51}{2}\right)$	M1	1.1b
	$\sum_{r=51}^{100} ln\left(\frac{r+1}{r-1}\right)^{35} = 35\,ln\left(\frac{100\times101}{2} \div \frac{50\times51}{2}\right)$	M1	3.1a
	$= 35\,ln\left(\frac{202}{51}\right)$	A1	1.1b
		(3)	
			(7 marks)

Notes:

(a)

M1: Uses the subtraction laws of logs to start the method of differences process.

dM1: Demonstrates the method of differences process, should have a minimum of e.g $r = 2$, $r = 3$, $r = 4$, $r = n - 1$ and $r = n$ shown -- enough to establish *at least one cancelling term* and *all non-disappearing terms* though the latter may be implied by correct extraction if only the first few cases are shown. Allow this mark if an extra term for $r = 1$ has been included.

A1: Correct terms that do not cancel - must not contradict their list of terms so e.g. if $r = 1$ was included, then A0A0 follows. The ln 1 may be included for this mark.

A1*: Achieves the printed answer, with no errors or omissions **and** must have had a complete list (as per dM1) before extraction (but condone missing brackets on ln terms). If working with r throughout, they must replace by n to gain the last A, but all other marks are available.

NB For attempts at combining log terms instead of using differences, full marks may be awarded for the equivalent steps, but attempts that do not make progress in combining terms will score no marks.

(b) Condone a bottom limit of 0 or 1 being used throughout part (b).

M1: Attempts to split into (the sum up to 100) – (the sum up to k) where k is 49, 50 or 51 **and** apply the result of (a) in some way. Condone slips with the power.

M1: Having attempted to apply (a), uses difference and power log laws correctly to reach an expression of the required form.

A1: Correct answer**.** Accept equivalents in required form, such as $35\,ln\,\frac{5050}{1275}$

Question	Scheme	Marks	AOs
5(a)	$det(\boldsymbol{M}) = a(6) - 2(4) - 3(2a - 12)$	M1	1.1b
	$det(\boldsymbol{M}) = 28 \neq 0$ therefore, non-singular for all values of a	A1	2.4
		(2)	
(b)	Finds the matrix of minors $\begin{pmatrix} 6 & 4 & 2a-12 \\ 4+3a & 2a+12 & a^2-8 \\ 9 & 6 & 3a-4 \end{pmatrix}$	M1	1.1b
	Finds the matrix of cofactors and transposes. $\begin{pmatrix} 6 & -4-3a & 9 \\ -4 & 2a+12 & -6 \\ 2a-12 & 8-a^2 & 3a-4 \end{pmatrix}$	M1	1.1b
	$\frac{1}{28}\begin{pmatrix} 6 & -4-3a & 9 \\ -4 & 2a+12 & -6 \\ 2a-12 & 8-a^2 & 3a-4 \end{pmatrix}$	M1 A1	1.1b 2.1
		(4)	
			(6 marks)

Notes:

(a)

M1: Finds the determinant of the matrix **M**. Must be seen in part (a). Allow one slip if no method shown.

A1: Correct value for determinant, states doesn't equal 0 (accept > 0) and draws the conclusion that the matrix is non-singular. If non-singular meaning determinant is non-zero is given in a preamble then accept a minimal conclusion (e.g. "hence shown"), but there must be a conclusion.

(b)

M1: Finds the matrix of minors, at least 5 correct values.

M1: Finds the matrix of cofactors and transposes (in either order). Note: some will do all these steps in one go, which is fine as long as it is clear what they have done. Allow minor slips if the process is clearly correct.

M1: Completes the process to find the inverse matrix, divides by the determinant.

A1: Correct matrix.

Question	Scheme	Marks	AOs
6(a)	$\frac{2x^2+3x+6}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} \Rightarrow 2x^2+3x+6$ $= A(x^2+4)+(Bx+C)(x+1)$	M1	1.1b
	e.g. $x=-1 \Rightarrow A = \ldots,\ x=0 \Rightarrow C = \ldots,\ \text{coeff } x^2 \Rightarrow B = \ldots$ or Compares coefficients and solves to find values for A, B and C $2 = A+B,\ 3 = B+C,\ 6 = 4A+C$	dM1	1.1b
	$A=1,\quad B=1,\quad C=2$	A1	1.1b
		(3)	
(b)	$\int_0^2 \frac{1}{x+1} + \frac{x+2}{x^2+4}\,dx = \int_0^2 \frac{1}{x+1} + \frac{x}{x^2+4} + \frac{2}{x^2+4}\,dx$ $= \left[\alpha\ln(x+1) + \beta\ln(x^2+4) + \lambda\arctan\left(\frac{x}{2}\right)\right]_0^2$	M1	3.1a
	$= \left[ln(x+1) + \frac{1}{2}ln(x^2+4) + arctan\left(\frac{x}{2}\right)\right]_0^2$	A1	2.1
	$= \left[ln(3) + \frac{1}{2}ln(8) + arctan\,1\right] - \left[ln(1) + \frac{1}{2}ln(4) + arctan(0)\right]$ $=$ $= \left[ln(3) + \frac{1}{2}ln(8) + arctan(1)\right] - \left[\frac{1}{2}ln\,4\right] = \underline{ln\left(\frac{3\sqrt{8}}{2}\right)} + \frac{\pi}{4}$	dM1	2.1
	$ln(3\sqrt{2}) + \frac{\pi}{4}$	A1	2.2a
		(4)	
		(7 marks)	

Notes:

(a)
M1: Selects the correct form for partial fractions and multiplies through to form suitable identity or uses a method to find at least one value (e.g. cover up rule).
dM1: Full method for finding values for all three constants. Dependent on first M. Allow slips as long as the intention is clear.
A1: Correct constants or partial fractions.

(b)
M1: Splits the integral into an integrable form and integrates at least two terms to the correct form**.** They may use a substitution on the arctan term
A1: Fully correct Integration.
dM1: Uses the limits of 0 and 2 (or appropriate for a substitution), subtracts the correct way round and combines the ln terms from separate integrals to a single term with evidence of correct ln laws at least once.
A1: Correct answer

Question	Scheme	Marks	AOs
7(a)	$z* = a - bi$ then $zz* = (a+bi)(a-bi) = \ldots$	M1	1.1b
	$zz* = a^2 + b^2$ therefore, a real number	A1	2.4
		(2)	
(b)	$\frac{z}{z*} = \frac{a+bi}{a-bi} = \frac{(a+bi)(a+bi)}{(a-bi)(a+bi)} = \frac{(a^2-b^2)+2abi}{a^2+b^2} = \frac{7}{9} + \frac{4\sqrt{2}i}{9}$ or $\frac{z}{z^*} = \frac{z^2}{zz^*} = \frac{z^2}{18} \Rightarrow$ $z^2 = 14 + 8\sqrt{2}i$ or $a + bi = \left(\frac{7}{9} + \frac{4\sqrt{2}i}{9}\right)(a - bi) = \ldots + \ldots i$	M1	1.1b
	Forms two equations from $a^2 + b^2 = 18$ or $\frac{a^2-b^2}{18} = \frac{7}{9}$ or $\frac{a^2-b^2}{a^2+b^2} = \frac{7}{9}$ or $\frac{2ab}{18} = \frac{4\sqrt{2}}{9}$ or $\frac{2ab}{a^2+b^2} = \frac{4\sqrt{2}}{9}$ or $a = \frac{7}{9}a + \frac{4\sqrt{2}}{9}b$ oe	M1 A1	3.1a 1.1b
	Solves the equations simultaneously e.g. $a^2 + b^2 = 18$ and $a^2 - b^2 = 14$ leading to a value for a or b	dM1	1.1b
	$z = \pm\left(4 + \sqrt{2}i\right)$	A1	2.2a
		(5)	
	(7 marks)		

Notes:

(a)(i)

M1: States or implies $z* = a - bi$ and finds an expression for $zz*$

A1: Achieves $zz* = a^2 + b^2$ and draws the conclusion that $zz*$ is a real number. Accept $\in \mathbb{R}$ as conclusion, but not just "no imaginary part".

(b)

M1: Starts the process of solving by using the conjugate to form an equation with real denominators, and without z^* or i^2 in the equation. Accept as shown in scheme, or may multiply through by $a - bi$ and expand and gather terms. May be implied by correct extraction of equation(s).

M1: Uses the given information to form two equations involving a and b at least one of which includes both. It must involve equating real or imaginary parts of $\frac{z}{z*} = \frac{7}{9} + \frac{4\sqrt{2}i}{9}$

A1: Any two correct equations arising from use of both given facts. (Note: if multiplying through by $a - bi$ then equating real and imaginary terms gives the same equation.)

dM1: Dependent on previous method mark, solves the equations to find a value for either a or b.

A1: Deduces the correct complex numbers and no extras. Do not accept $\pm 4 \pm \sqrt{2}i$

Note: it is possible to solve via polar coordinates, but unlikely to succeed. If you see responses you think are worthy of credit but are unsure how to mark, use review. Example solutions shown below.

(b) Alt	$\frac{z}{z^*} = \frac{z^2}{zz^*} = \frac{z^2}{18} \Rightarrow z^2 = 14 + 8\sqrt{2}i$ or let arg $z = \theta$. then $\frac{z}{z*} = \frac{re^{i\theta}}{re^{-i\theta}} = e^{2i\theta} = cos\,2\,\theta + \text{i}\sin 2\theta$	M1	1.1b
	$z^2 = 18(cos\,\alpha + \text{i}\sin\alpha)$where $\tan\alpha = \frac{4\sqrt{2}}{7} \Rightarrow z = \pm\sqrt{18}\left(cos\frac{1}{2}\alpha + \text{i}\sin\frac{1}{2}\alpha\right)$Or $cos\,2\,\theta + \text{i}\sin 2\theta = \frac{7}{9} + \frac{4\sqrt{2}i}{9} \Rightarrow 2\,cos^2\,\theta - 1 = \frac{7}{9}, 2\,sin\,\theta\,cos\,\theta = \frac{4\sqrt{2}}{9}$	M1 A1	1.1b 1.1b
	$cos\frac{1}{2}\alpha = \sqrt{\frac{1}{2}(1 + cos\,\alpha)} = \sqrt{\frac{1}{2}\left(1 + \frac{7}{9}\right)} =\ldots$ and $\sin\frac{1}{2}\alpha = \sqrt{\frac{1}{2}(1 - cos\,\alpha)} = \sqrt{\frac{1}{2}\left(1 - \frac{7}{9}\right)} =\ldots$or $\Rightarrow cos\,\theta = \frac{2\sqrt{2}}{3}, sin\,\theta = \frac{1}{3}, r = \|z\| = \sqrt{zz^*} = \sqrt{18}$	dM1	3.1a
	$z = \pm\left(4 + \sqrt{2}i\right)$	A1	2.2a
		(5)	

Question	Scheme	Marks	AOs
8(a)	$\left(z+\frac{1}{z}\right)^6 = 64\cos^6\theta$	B1	2.1
	$\left(z+\frac{1}{z}\right)^6 = z^6 + 6(z^5)\left(\frac{1}{z}\right) + 15(z^4)\left(\frac{1}{z^2}\right) + 20(z^3)\left(\frac{1}{z^3}\right) + 15(z^2)\left(\frac{1}{z^4}\right) + 6(z)\left(\frac{1}{z^5}\right) + \left(\frac{1}{z^6}\right)$	M1	2.1
	$= \left[z^6 + \frac{1}{z^6}\right] + 6\left[z^4 + \frac{1}{z^4}\right] + 15\left[z^2 + \frac{1}{z^2}\right] + 20$	A1	1.1b
	Uses $z^n + \frac{1}{z^n} = 2\cos n\theta$ $\{64\cos^6\theta\} = 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$	M1	2.1
	$32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$ * cso	A1 *	1.1b
		(5)	
(b)	$\boldsymbol{H = 2}$	B1	3.3
		(1)	
(c)	$\text{vol} = \left\{\frac{1}{2}\right\}\pi\int\left(2\cos^3\left(\frac{x}{4}\right)\right)^2 \,\mathrm{d}x$	B1ft	3.4
	$\text{vol} = \{2\pi\}\int \cos^6\left(\frac{x}{4}\right)\mathrm{d}x$ $= \{2\pi\}\int \frac{1}{32}\left(\cos\left(\frac{6x}{4}\right) + 6\cos\left(\frac{4x}{4}\right) + 15\cos\left(\frac{2x}{4}\right) + 10\right)\mathrm{d}x = \ldots$	M1	1.1b
	$= \{2\pi\}\left[\frac{1}{32}\underline{\left(\frac{2}{3}\sin\left(\frac{3x}{2}\right) + 6\sin(x) + 30\sin\left(\frac{x}{2}\right) + 10x\right)}\right]$	A1	1.1b
	$= 2\times 2\pi\left[\frac{1}{32}\left(\frac{2}{3}\sin\left(\frac{3}{2}\times 4\right) + 6\sin(4) + 30\sin\left(\frac{4}{2}\right) + (10\times 4)\right) - 0\right] = \ldots$ or $=$ $2\pi\left[\frac{1}{32}\left(\frac{2}{3}\sin\left(\frac{3}{2}\times 4\right) + 6\sin(4) + 30\sin\left(\frac{4}{2}\right) + (10\times 4)\right) - \frac{1}{32}\left(\frac{2}{3}\sin\left(\frac{3}{2}\times -4\right) + 6\sin(-4) + 30\sin\left(-\frac{4}{2}\right) + (10\times -4)\right)\right]$ …	dM1	3.4
	$= 24.56$	A1	1.1b
		(5)	

(d)	The equation of the curve may not be suitable The measurements may not be accurate The paperweight may not be smooth	B1	3.5b
		(1)	
			(12 marks)

Notes:

(a)

B1: Correct identity or equivalent rearrangement. This can appear anywhere in the proof.

M1: Attempts the expansion of $\left(z+\frac{1}{z}\right)^6$ must have at least 3 correct terms. Combining the powers when expanding is fine.

A1: Correct expansion with z terms simplified, need not be rearranged. (So a correct expansion will score M1A1.)

M1: Uses $z^n + \frac{1}{z^n} = 2\cos n\,\theta$ to write the expression in terms multiple angles of $\cos 6\,\theta,\ \cos 4\,\theta$ and $\cos 2\,\theta$. Pairing of terms must be seen.

A1*: Achieves the printed answer with no errors or omissions. Cso

For approaches using De Moivre B0M1A1M0A0 may be scored if the binomial expansions is attempted (and correct for the A).

(b)

B1: See scheme

(c)

Note: The question instructs use of algebraic integration and part (a), so answer only can score at most B1 for implied correct formula.

B1ft: Correct expression for the volume of the paperweight **or** the solid formed through 360º rotation, stated or implied, ignore limits. No need to expand, but must be applied, not just a formula in *y*, though allow a correct formula followed by correct integral if the π disappears. Follow through their *H*

M1: Uses the result in part part (a) to express the volume in an integrable form and attempts to integrate. Note use of *θ* instead of *x* is permissible for this mark. Allow if one term is missing or miscopied.

A1: Correct integration in terms of *x*. Ignore π, their H^2 and the $\frac{1}{32}$. Note if *θ* has been used it is A0 unless a correct substitution method has been implied as the coefficients will be incorrect.

dM1: Dependent on previous method mark and must have reached and integral of the correct form -- in terms of *x* with correct arguments allowing for one slip. Finds the required volume using either $\pi\int_0^4 y^2\,\mathrm{d}x$ or $\frac{1}{2}\pi\int_{-4}^4 y^2\,\mathrm{d}x$ and applies their limits - accept any value following a valid attempt at the integration as an attempt at applying limits.

A1: cao 24.56

(d)

B1: States an appropriate limitation. See scheme for some examples. The limitation should refer to the paperweight, not to paper. Do not accept “it does not take into account thickness of material” as it is a solid, not a shell, being modelled. Award the mark for a correct reason if two reasons are given and one is incorrect.

Question	Scheme	Marks	AOs
9(i) (a)	E. g. ● Because the interval being integrated over is unbounded. ● cosh x is undefined at the limit of ∞ ● the upper limit is infinite	B1	1.2
		(1)	
(i) (b)	$\int_0^\infty \cosh x \, dx = \lim_{t\to\infty} \int_0^t \cosh x \, dx$ or $\lim_{t\to\infty} \int_0^t \frac{1}{2}(e^x + e^{-x})dx$	B1	2.5
	$\int_0^t \cosh x \, dx = [\sinh x]_0^t = \sinh t \; (-0)$ or $\frac{1}{2}\int_0^t e^x + e^{-x} \, dx = \frac{1}{2}[e^x - e^{-x}]_0^t = \frac{1}{2}[e^t - e^{-t}]\left(-\frac{1}{2}[e^0 - e^0]\right)$	M1	1.1b
	When $t \to \infty$ $e^t \to \infty$and $e^{-t} \to 0$ therefore the integral is divergent	A1	2.4
		(3)	
(ii)	$4\,sinh\,x = p\,cosh\,x \Rightarrow tanh\,x = \frac{p}{4}$ or $4\,tanh\,x = p$ **Alternative** $\frac{4}{2}(e^x - e^{-x}) = \frac{p}{2}(e^x + e^{-x}) \Rightarrow 4e^x - 4e^{-x} = pe^x + pe^{-x}$ $e^{2x}(4-p) = p + 4 \Rightarrow e^{2x} = \frac{p+4}{4-p}$	M1	3.1a
	$\left\{-1 < \frac{p}{4} < 1 \Rightarrow\right\} -4 < p < 4$	A1	2.2a
		(2)	
		(6 marks)	

(i)(a)

B1: For a suitable explanation. Technically this should refer to the interval being unbounded, but this is unlikely to be seen. Accept "Because the upper limit is infinity", but **not** "because it is infinity" without reference to what "it" is. Do not accept "the upper limit tends to infinity" or "the integral is unbounded".

(i)(b)

B1: Writes the integral in terms of a limit as $t \to \infty$ (or other variable) with limits 0 and "t", or implies the integral is a limit by subsequent working by correct language.

M1: Integrates cosh$x$ correctly either as sinh x or in terms of exponentials and applies correctly the limits of 0 and "t". The bottom limit zero may be implied. No need for the $\lim_{t\to\infty}$ for this mark but substitution of ∞ is M0.

A1: cso States that (as $t \to \infty$) $sinh\,t \to \infty$ or $e^t \to \infty$and $e^{-t} \to 0$ therefore divergent (or not convergent), or equivalent working. Accept sinh t is undefined as $t \to \infty$

(ii)

M1: Divides through by cosh$x$ to find an expression involving tanh x

Alternative: uses the correct exponential definitions and finds an expression for e^{2x} or solves a quadratic in e^{2x}

A1: Deduces the correct inequality for p. Note $|p| < 4$ is a correct inequality for p.

uestion	Scheme		Marks	AOs
10(a)(i)	$\frac{d\theta}{dt} = \alpha \sin 3t + \beta t \cos 3t$ and $\frac{d^2\theta}{dt^2} = \delta \cos 3t + \gamma t \sin 3t$	Let $\theta = \lambda t \sin 3t$ $\frac{d\theta}{dt} = \alpha \sin 3t + \beta t \cos 3t$ and $\frac{d^2\theta}{dt^2} = \delta \cos 3t + \gamma t \sin 3t$	M1	1.1b
	$\frac{d\theta}{dt} = \frac{1}{12}\sin 3t + \frac{1}{4}t\cos 3t$ and $\frac{d^2\theta}{dt^2} = \frac{1}{4}\cos 3t + \frac{1}{4}\cos 3t - \frac{3}{4}t\sin 3t$ $= \frac{1}{2}\cos 3t - \frac{3}{4}t\sin 3t$	$\frac{d\theta}{dt} = \lambda \sin 3t + 3\lambda t \cos 3t$ and $\frac{d^2\theta}{dt^2} = 3\lambda\cos 3t + 3\lambda\cos 3t - 9\lambda t\sin 3t$ $= 6\lambda\cos 3t - 9\lambda t \sin 3t$	A1	1.1b
	$\frac{1}{2}\cos 3t - \frac{3}{4}t\sin 3t + 9\left(\frac{1}{12}t\sin 3t\right) = \ldots$	$6\lambda\cos 3t - 9\lambda t\sin 3t + 9(\lambda t\sin 3t) = \frac{1}{2}\cos 3t \Rightarrow \lambda = \ldots$	dM1	3.4
	$= \frac{1}{2}\cos 3t$ so PI is $\theta = \frac{1}{12}t\sin 3t$ *	$\theta = \frac{1}{12}t\sin 3t$ *	A1*	2.1
			(4)	
(a)(ii)	$m^2 + 9 = 0 \Rightarrow m = \pm 3i$		M1	1.1b
	$\theta = A\cos 3t + B\sin 3t$		A1	1.1b
	$(\theta =)CF + PI$		dM1	1.1b
	$\theta = A\cos 3t + B\sin 3t + \frac{1}{12}t\sin 3t$		A1	1.1b
			(4)	
(b)	$t = 0,\ \theta = \frac{\pi}{3} \Rightarrow A = \ldots\left\{\frac{\pi}{3}\right\}$		M1	3.4
	$t = 0,\ \frac{d\theta}{dt} = -3A\sin 3t + 3B\cos 3t + \frac{1}{12}\sin 3t + \frac{1}{4}t\cos 3t = 0$ $\Rightarrow B = \ldots\{0\}$		M1	3.4
	$\alpha = \frac{\pi}{3}\cos(3\times 10) + \frac{1}{12}(10)\sin(3\times 10) = \ldots$		ddM1	1.1b
	$\alpha = \pm$awrt 0.662		A1	3.4
			(4)	
(c)	0.662 is close to 0.62 so a good model (at t = 10)		B1ft	3.5a
			(1)	
(d)	$\frac{d^2\theta}{dt^2} + 9\theta = 0$ oe		B1	3.5c
			(1)	

	(14 marks)
Notes:	

(a)(i) Note: mark (a) as a whole

M1: Differentiates the given PI twice using the product rule to achieve the required form.

Alternatively, uses a correct form for the PI and differentiates twice using the product rule to achieve the required form. A correct form may involve other terms with coefficients that will be zero, e.g. $\theta = \lambda t \sin 3t + \mu t \cos 3t$ is fine. Also allow e.g $\theta = \lambda t \sin \quad \omega t$

A1: Correct derivatives.

dM1: Depends on first M, substitutes into the given differential equation and attempts to simplify. In the Alt they must go on to find value for λ .

A1*: Achieves $\frac{1}{2}\cos 3t$and makes a minimal conclusion (e.g //). Alternatively reaches the correct PI.

(a)(ii)

M1: Uses the model to form and solve the auxiliary equation. Accept $m^2 + 9 = 0 \rightarrow m = \pm 3i$ or ± 3

A1: Correct complementary function. Must be in terms of *t* but allow recovery if initially in terms of *x* but changed later.

dM1: Dependent on the previous method mark. Finds the general solution by adding the particular integral to the complementary function.

A1: Correct general solution including "$\theta =$ ", which may be recovered in part (b).

(b)

M1: Uses the initial conditions of the model, $t = 0,\ \theta = \frac{\pi}{3}$to find a value for a constant.

M1: Differentiates the general solution and uses the initial conditions of the model $t = 0,\ \frac{d\theta}{dt} = 0$ to find a value for the other constant.

ddM1: Dependent on both previous method marks. Substitutes *t* = 10 into their particular solution. If not substitution is seen, accept any value as the attempt as long as they have found all relevant constants.

A1: Accept awrt ±0.662

(c)

B1ft: Makes a quantitative comparison of the size of their answer to part (b) with 0.62 and makes conclusion (e.g. good model). Follow through on their answer to (b) and draws an appropriate conclusion about the model. Accept “not reasonable” as long as it is supported with evidence but there must be some instructive comparison and a conclusion about the model - not just stating how much it is out. The reason given must be correct.

Accept e.g. a correct percentage error with reasonable conclusion, or statement approximately equal with conclusion.

Do not accept e.g. “does not agree to 1 s.f.” or “out by 0.6” as these lacks context. Do not accept arguments based solely on a difference in sign, they must be referring to the relative size of angle.

(d)

B1: Refines the model, accept any constant on the right hand side.

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